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STRESS STATE OF A STRAIGHT ISOLATED CUT, LOADED FROM WITHOUT BY CONCENTRATED FORCES AND GROWING AT A CONSTANT RATE

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In [1] an investigation was made of the stress state of a straight isolated cut, developing at a constant rate under the conditions of antiplane deformation of the ideal theory of elasticity. Here, we consider cases of total self-similar loading and non-self-similar loading by concentrated forces, applied at the middle of the cut to its edges and depending arbitrarily on the time.

In the present work an analogous investigation is made within the framework of the ideal theory of elasticity for the case of plane deformation; here, use was made of results and ideas published in [2, 3].

In the unloaded elastic xy plane, at the initial moment of time t = 0, let a cut loaded by forces concentrated along its edges start to develop along the x axis from the origin of coordinates with the rate v. It is required to determine the stress state arising in the plane, specifically, the value of the coefficient of the field intensity of the stresses near the tip of the cut. The elastic displacements, as is well known [4], satisfy the following equations:

$$w_i = u_i + v_i, \quad \Delta u_i = \frac{1}{a^2}\frac{\partial^2 u_i}{\partial t^2}, \quad \Delta v_i = \frac{1}{b^2}\frac{\partial^2 v_i}{\partial t^2}, \quad \frac{\partial u_1}{\partial y} = \frac{\partial u_2}{\partial x}, \quad \frac{\partial v_1}{\partial x} = -\frac{\partial v_2}{\partial y}, \tag{0.1}$$

where $u_i(x, y, t)$, $v_i(x, y, t)$ are the potential and solenoidal components of the displacement vector w_i (x, y, t); a and b are the velocities of the longidinal and transverse waves of the elastic plane.

The components of the stress tensors are expressed in terms of the displacements by the formulas

$$\sigma_{xx} = \mu\left[\frac{a^2}{b^2}\left(\frac{\partial w_1}{\partial x} + \frac{\partial w_2}{\partial y}\right) - 2\frac{\partial w_2}{\partial y}\right], \quad \sigma_{yy} = \mu\left[\frac{a^2}{b^2}\left(\frac{\partial w_1}{\partial x} + \frac{\partial w_2}{\partial y}\right) - 2\frac{\partial w_1}{\partial x}\right], \quad \sigma_{xy} = \mu\left[\frac{\partial w_1}{\partial y} + \frac{\partial w_2}{\partial x}\right]. \tag{0.2}$$

We consider the region of the upper half plane y > 0, bounded by the arc of the longitudinal wave, emitted at the initial moment of time (Fig. 1). In this region a solution of system (0.1) is sought, satisfying the following boundary conditions. At the edge of the cut, with $|x| < vt$, the external load is given: $\sigma_{yy} = -\sigma_y(x, t)$; $\sigma_{xy} = 0$. The form of the function $\sigma_y(x, t)$ will be refined in what follows.

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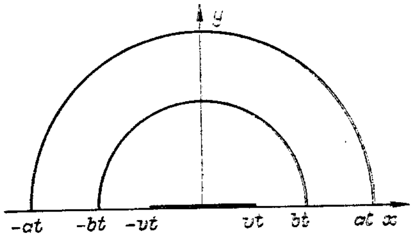

Fig. 1

In the sections of the x axis from the tip of the cut to the longitudinal wave $vt < |x| < at$, $y = 0$, satisfaction of the conditions is required: $w_2 = 0$, $\sigma_{xy} = 0$, flowing out of the assumed symmetry of the problem with respect to the $y = 0$ axis. The initial conditions of the problem are null conditions. A solution is sought, having radical singularities for the stresses at the tips of the cut, near which along the x axis the component of the stress tensor σ_{yy} has the form

$$\sigma_{yy}(x, 0, t)|_{x\to vt} \approx \frac{2\pi N(t)}{\sqrt{2\pi(x - vt)}}, \tag{0.3}$$

where N(t) is the sought coefficient of the stress intensity.

1. Self-Similar Stress. We consider the case of stress with which the external load has the form

$$\sigma_y = \frac{Qt^{l-1}}{at_0^{l-1}}\sigma\left(\frac{x}{at}\right), \tag{1.1}$$

where $Q(at_0, t_0)$ are the constants of dimensionality of the pressure and the time, l is a whole positive number. In this case, the functions

$$U_i = \partial^l u_i/\partial t^l,\ V_i = \partial^l v_i/\partial t^l$$

are homogeneous zero-order functions with respect to the variables x, y, t. Under these circumstances, U_i satisfy the wave equation for longitudinal waves, V_i for transverse waves. In accordance with [4], they can be represented as the real parts of analytical functions in the complex planes z_1 and z_2:

$$U_i(x/at, y/at) = \operatorname{Re} U_i^l(z_1), \quad V_i(x/at, y/at) = \operatorname{Re} V_i^l(z_2),$$

where z_1 and z_2 are connected with the variables x, y, t by the relationships

$$\delta_1 \equiv t - z_1 x - y\sqrt{a^{-2} - z_1^2} = 0, \quad \delta_2 \equiv t - z_2 x - y\sqrt{b^{-2} - z_2^2} = 0. \tag{1.2}$$

Specifically, with $y = 0$, $z_1 = z_2 = t/x$. The branches of the radicals in planes with cuts, connecting $(a^{-1}, -a^{-1})$, $(b^{-1}, -b^{-1})$ are chosen that, with $z \to \infty$, they tend toward $i\sqrt{z_{1,2}}$.

We introduce the analytical function W(z), connected with the sought functions in the following way:

$$W(z) = U_2^l(z) + V_2^l(z).$$

The last two relationships of system (0.1) and the condition $\sigma_{xy} = 0$ with $y = 0$ can be satisfied if it is assumed that [3]

$$(U_1^l)'_z = \frac{b^{-2} - 2z^2}{b^{-2}\sqrt{a^{-2} - z^2}} W'_z, \quad (U_2^l)'_z = -\frac{2z\sqrt{b^{-2} - z^2}}{b^{-2}} W'_z, \tag{1.3}$$

$$(V_1^l)'_z = \frac{b^{-2} - 2z^2}{b^{-2}} W'_z, \quad (V_2^l)'_z = \frac{2z^2}{b^{-2}} W'_z.$$

Here, at the y = 0 axis (0.2)

$$\frac{\partial^l \sigma_{yy}}{\partial t^l} = \frac{\mu}{t} \operatorname{Re}\left[- \frac{S(z) z W'_z}{b^{-2} \sqrt{a^{-2} - z^2}} \right], \tag{1.4}$$
$$S(z) = (b^{-2} - 2z^2)^2 + 4z^2 \sqrt{a^{-2} - z^2} \sqrt{b^{-2} - z^2}.$$

In the z plane we state the boundary-value problem for the function W'_z. At the edges of the cut with $|x| < vt$, $y = 0$, which, in accordance with (1.2), in the z plane corresponds to the interval $|\operatorname{Re} z| > v^{-1}$, $\operatorname{Im} z = 0$, the condition is satisfied

$$\operatorname{Im} W'_z = \frac{t b^{-2} \sqrt{z^2 - a^{-2}}}{\mu z S(z)} \frac{\partial^l \sigma_y}{\partial t^l}. \tag{1.5}$$

With $b^{-1} < |\operatorname{Re} z| < v^{-1}$, $\operatorname{Im} z = 0$, which, in the physical xy plane corresponds to the sections from the tips of the cut to the transverse wave, we have $w_2(x, 0, t) = 0$ and, consequently, $\operatorname{Re} W(z) = 0$. From this, $\operatorname{Re} W'_z = 0$. From the condition of the equality to zero of the vertical displacements at the y = 0 axis in a longitudinal wave with $bt < |x| < at$, and the inequalities flowing out of the null zero conditions

$$\operatorname{Re} U_i^l(z) = 0 \text{ with } |\operatorname{Re} z| < a^{-1}, \ \operatorname{Im} z = 0,$$
$$\operatorname{Re} V_i^l(z) = 0 \text{ with } |\operatorname{Re} z| < b^{-1}, \ \operatorname{Im} z = 0$$

it follows that, with $|\operatorname{Re} z| < b^{-1}$, $\operatorname{Im} z = 0$, the condition $\operatorname{Re} W'_z = 0$ is also satisfied.

Thus, in the lower half plane z we have a mixed Keldysn—Sedov problem for the function W'_z. We shall seek its solution in the upper half plane of the $z_3 = 1/az$ plane.

From requirement (0.3) with respect to the behavior of the stresses at the tips of the cuts it follows that, with $z_3 = \pm v/a$, $z = \pm v^{-1}$ $W'_z \sim (z_3 - v^2/a^2)^{-(l+1/2)}$. As in [1], at the intersection points of the x, y axes of the physical plane with the longitudinal and transverse waves, as well as at the origin of coordinates, the condition of regularity of function W(z) must be satisfied

$$W \sim a_0 + a_1(z_3 \pm b/a) + \ldots, \quad W'_z \sim a_1 \text{ with } z_3 \to \pm b/a,$$
$$W \sim a_0 + a_1(z_3 \pm 1) + \ldots, \quad W'_z \sim a_1 \text{ with } z_3 \to \pm 1,$$
$$W \sim i(a_1 z + a_3 z^3 + \ldots), \quad W'_z \sim i a_1 \text{ with } z_3 \to \pm \infty,$$
$$W \sim a_0 + a_{-2}/z^2 + \ldots, \quad W'_z \sim z_3^3 \text{ with } z_3 \to 0.$$

The last two conditions take account of the symmetry of the problem with respect to the x = 0 axis in the physical plane, postulated in what follows.

The function W'_z, having these properties and satisfying the boundary conditions at the $\operatorname{Im} z_3 = 0$ axis, has the form

$$W'_z = \frac{z_3^2}{(z_3^2 - v^2/a^2)^{l+1/2}} \left[\frac{1}{\pi} \int_{-v/a}^{v/a} \frac{(s^2 - v^2/a^2)^{l+1/2} \operatorname{Im} W'_z ds}{s^2 (s - z_3)} + i z_3 \sum_{i=0}^{l-1} A_i z_3^{2i} \right], \tag{1.6}$$

Im W'here is defined in (1.5). The solution obtained contains l constants A_i, whose values can be obtained, e.g., from the system of equations

$$\frac{\partial^r \sigma_{yy}}{\partial t^r}(vt - 0, 0, t) = - \frac{\partial^r \sigma_y}{\partial t^r}(vt, 0, t), \quad r = 0, \ldots, l-1. \tag{1.7}$$

Here σ_{yy} must be expressed in terms of solution (1.6) using (1.4); σ_y is given in (1.3).

In what follows we will examine in more detail the partial case of loading (1.3), i.e., the case of a load concentrated at the origin of coordinates

$$\sigma_y = Qt^l \delta_1(x)/t_0^l, \tag{1.8}$$

where Q, t_0 are the constants of the dimensionality of the force for unit length and time. In this case

$$\frac{\partial^l \sigma_y}{\partial t^l} = \frac{Ql!\delta_1(x_3)}{t_0^l at} = \frac{f(x_3)}{at},$$

where $x_3 = x/at$. From (1.5) we obtain

$$\operatorname{Im} W_z' = \frac{x_3^4 a^3 \sqrt{1 - x_3^2}\, f(x_3)}{\mu b^2 D(x_3)},$$

$$D(x_3) = a^4 x_3^4 S(1/ax_3) = [2 - (a/b)^2 x_3^2]^2 - 4\sqrt{1 - x_3^2}\sqrt{1 - (a/b)^2 x_3^2}.$$

Substituting the expression obtained for Im W_z' into (1.6), we obtain

$$W_z' = \frac{iCz^{2l}}{(v^{-2} - z^2)^{l+1/2}}\left[1 + \frac{1}{z^2}\sum_{i=0}^{l-1} A_i z^{-2i}\right], \quad C = \frac{(-1)^l a^2 l! Q}{2\pi\mu b^2 t_0^l (1 - a^2/b^2)}. \tag{1.9}$$

Integrating (1.4) with respect to t, with y = 0 we obtain

$$\frac{\partial^r \sigma_{yy}}{\partial t^r} = \mu \frac{x^{l-1-r}}{(l-1-r)!} \operatorname{Re} \int_{a^{-1}}^{z} (z - s)^{l-1-r} f_1(s)\, ds, \quad r = 0, \ldots, l-1, \tag{1.10}$$

$$f_1(s) = -\frac{S(s) W_z'}{b^{-2}\sqrt{a^{-2} - s^2}}.$$

Substituting these expressions into (1.7), taking account of (1.8) and (1.9), for determination of the constants A_i in (1.9) we obtain the system

$$\operatorname{Re} \int_{a^{-1}}^{v^{-1}+0} (v^{-1} - s)^{l-1-r} f_2(s)\, ds = 0, \quad r = 0, \ldots, l-1,$$

$$f_2(s) = -\frac{s^{2l-1} C S(s)}{(v^{-2} - s^2)^{l+1/2} b^{-2} \sqrt{s^2 - a^{-2}}}\left(1 + \frac{1}{s^2}\sum_{i=0}^{l-1} A_i s^{-2i}\right).$$

After identical transformations, this system is brought to the following:

$$\sum_{i=0}^{l-1} \bar{A}_i B_{ji} + B_{jl} = 0, \quad j = 0, \ldots, l-1, \tag{1.11}$$

$$B_{ji} = \operatorname{Re} \int_{1}^{\bar{v}^{-1}+0} \frac{z^{3l-2i-j-3}\bar{S}(z)\, dz}{\sqrt{z^2 - 1}\,(\bar{v}^{-2} - z^2)^{l+1/2}}, \quad B_{jl} = B_{ji}|_{i=-1},$$

$$\bar{S}(z) = (a^2/b^2 - 2z^2)^2 + 4z^2\sqrt{1 - z^2}\,\sqrt{a^2/b^2 - z^2}.$$

Here the notation is introduced: $\bar{v} = v/a$, $\bar{A}_i = A_i a^{2(i+1)}$. The integration path with numerical calculations in integrals (1.11) was selected going around the point $\bar{v}^{-1}$ along the arc of a circle with its center at this point.

The expression for the coefficient of the stress intensity (0.3) can be found, using (1.9), from (1.10) with $r = l - 1$.

Here we obtain

$$N(t) = N^0(t)\,R(\bar{v}, l), \quad N^0(t) = \frac{2\pi Q t^l}{t_0^l \sqrt{\pi v t}}, \tag{1.12}$$

$$R(\bar{v}, l) = \frac{(-1)^l \bar{v}^2 l \Sigma \bar{S}(\bar{v}^{-1})\left(1 + \bar{v}^2 \sum_{i=0}^{l-1} \bar{A}_i \bar{v}^{2i}\right)}{2^l (1 - a^2/b^2)\sqrt{1 - \bar{v}^2}},$$

$$\sum = 1 - C_1^{l-1}/3 + C_2^{l-1}/5 - \ldots (-1)^{l-1}/(2l-1), \quad C_m^n = \frac{n!}{m!\,(n-m)!}.$$

The value of $N^\circ(t)$ is equal to the value of the coefficient of the stress intensity, arising in the static problem of the loading of a motionless cut with a length of 2vt by the load $\sigma_y = Q(t/t_0)^l \delta_1(x)$. To find $R(\bar{v}, l)$, in (1.12) the following procedure was used. For the selected values of $\bar{v}$ and l, the coefficients of the matrix of the system (1.11) were calculated. Its solution determined the constants $\bar{A}_i$ and the value of $R(\bar{v}, l)$.

Thus, the values of $R(\bar{v}, l)$ were found for $l = 0, \ldots, 9$, v/b = 0.1, 0.2,...,0.8 with $b/a = 0.6$. These values are given in Table 1.

For $l = 0$, the function $R(\bar{v}, 0)$ was calculated using the formula [3]

$$R(\bar{v}, 0) = \frac{\bar{v}^2 \bar{S}(\bar{v}^{-1})}{2\sqrt{1 - \bar{v}^2}\,[1 - (a/b)^2]}, \tag{1.13}$$

which is obtained also from the general solution of (1.9), if, in it, it is assumed that $l = 0$, $A_i = 0$.

In the case $l = 1$, to find the one unknown constant, we have one linear equation, whose coefficients can be found analytically in terms of elliptical integrals

$$R(\bar{v}, 1) = \frac{\bar{v}^4 I_1 \bar{S}(\bar{v}^{-1})}{2 I_2 [1 - (a/b)^2]\sqrt{1 - \bar{v}^2}}, \tag{1.14}$$

$$I_1 = \operatorname{Re}\int_1^{\bar{v}^{-1}} \frac{\bar{S}(z)\,dz}{\sqrt{z^2 - 1}\sqrt{\bar{v}^{-2} - z^2}} = \frac{\bar{v}^{-3}}{3}\left[4(2\bar{v}^2 - 3m^2 + 2)E(q) + \right.$$

$$\left. + (3m^4 - 4\bar{v}^2)F(q) + 4(m^2 - 2)E(q') + m^2 F(q')\right],$$

$$I_2 = \operatorname{Re}\int_1^{\bar{v}^{-1}+0} \frac{\bar{S}(z)\,dz}{\sqrt{z^2 - 1}\,(\bar{v}^{-2} - z^2)^{3/2}} = \frac{\bar{v}^{-1}}{1 - \bar{v}^2}\{[m^4 + 4\bar{v}^2(1 - m^2)]F(q) -$$

$$- [(2 - m^2)^2 + 4(1 - \bar{v}^2)]E(q) + 8(1 - \bar{v}^2)E(q') - 4m^2(1 - \bar{v}^2)F(q')\},$$

$$q = \sqrt{1 - \bar{v}^2}, \quad q' = \sqrt{1 - m^2}, \quad m = v/b,$$

F(q), E(q) are total elliptical integrals of the first and second kinds, respectively.

The results of calculations, carried out using this formula, coincided with those obtained using the above method, to the required degree of exactness.

An interesting special characteristic of the problem under consideration in comparison with its antiplane variant is the fact that, for some interval of rates of propagation of the cut greater than the critical v*, the function $R(\bar{v}, l)$ takes on a negative value. Here the value of the velocity v*, with which the function $R(\bar{v}, l)$ goes over into the negative region, decreases with a rise in l. Table 2 gives the value of the velocity v^*/b for $1 \leqslant l \leqslant 9$, $b/a = 0.6$.

2. <u>Non-Self-Similar Concentrated Loading.</u> The case of loading with which $\sigma_y = \sigma(t) \times \delta_1(x)$ with an arbitrary function $\sigma(t)$ is of great practical importance. It can be approximately considered [5] using self-similar solutions of Sec. 1 if we represent the function $\sigma(t)$ in the form of a segment of the series

TABLE 1

l \ v/b	0,1	0,2	0,3	0,4	0,5	0,6	0,7	0,8
0	0,9923644	0,9691741	0,9295275	0,8717291	0,7929041	0,6881701	0,5486786	0,3560028
1	0,9396547	0,8100651	0,6491022	0,48011165	0,3190632	0,177724237	0,06631309	—0,003102074
2	0,8686700	0,6399174	0,4172112	0,2374483	0,1097657	0,031654	—0,004597291	—0,01011845
3	0,7898806	0,4900660	0,2574812	0,10993404	0,03090074	—0,001638966	—0,00783898	—0,00385634
4	0,709747	0,3683777	0,1544932	0,0468369	0,00406344	—0,00612688	—0,0044095	—0,001393177
5	0,6321809	0,273437	0,0900969	0,0169753	—0,0035219	—0,00473747	—0,00214678	—0,000565843
6	0,559399	0,200957	0,0506253	0,0036505	—0,0045798	—0,00295714	—0,0010408	—0,000258584
7	0,492500	0,146337	0,026877	—0,0017094	—0,00379115	—0,00173	—0,00052476	—0,00012716
8	0,43192	0,10554	0,0129035	—0,003398	—0,0027368	—0,00099965	—0,0002769	—0,00006475
9	0,3775	0,0752	0,004922	—0,003514	—0,001868	—0,00058193	—0,000151	—0,00003395

TABLE 2

l	1	2	3	4	5	6	7	8	9
v^*/b	0,8	0,69	0,59	0,54	0,48	0,44	0,39	0,38	0,36

$$\sigma(t) = \sum_{i=0}^{r} a_i t^i \tag{2.1}$$

in the interval 0 < t < T.

In this case, for the coefficient of the stress intensity we obtain the expression

$$N(t) = \frac{1}{\sqrt{\pi v t}} \sum_{i=0}^{r} a_i R(\bar{v}, i)\, t^i. \tag{2.2}$$

The coefficients a_i for $\sigma(t) \in L_2$ can be found, e.g., from the representation of σ(t) in the interval (0, T) in the form of an expansion in terms of Legendre polynomials.

The above procedure was carried through for two laws of change in the load

$$\sigma_1(t) = \delta_0(t)\ \delta_0(1-t),\ 0 < t < T,$$

$$\sigma_2(t) = \delta_0(t)\delta_0(1-t)\sin \pi t,\ 0 < t < T.$$

Here $\delta_0(t)$ is a Heaviside function. An investigation of the effect of r on the degree of approach of series (2.1) to the exact value of function σ(t) and on the convergence of N(t) with respect to r (2.2) showed that a change in r from 5 to 9 brings about only a slight change in the value of σ(t) and N(t), calculated using formulas (2.1) and (2.2). It was also remarked that the adopted polynomial representation well approximates a continuous law, while, at the same time, for a discontinuity in the region, there are considerable deviations.

Figures 2 and 3 show the dependences N(t) for the first loading law with v/b = 0.2, 0.4. The dashed and dash—dot lines in Figs. 2 and 3 correspond to values T = 2.4. The calculations were made with r = 9. The solid lines correspond to dependences N(t), obtained for the above case from a solution [2] of the problem of the stress state of a plane with an arbitrarily growing isolated cut, loaded by an arbitrarily varying load. In the general case, this solution is contained in the expression for the coefficient of stress intensity as the minimum of triple integrals, but, in the case of the law under consideration $\sigma_y(x, t) = \delta_0(t)\delta_0(1-t)\delta_1(x)$, this solution is simplified and can be represented in the form

$$N(t) = \begin{cases} N_0, & 0 < t < t_0 = (1 - v/a)^{-1}, \\ N_0 + mI_2, & t_0 < t < t_1 = (1 - v/b)^{-1}, \\ N_0 - m(I_0 - I_1), & t_1 < t < t_2 = (1 - v/c)^{-1}, \\ N_0 - mI_0, & t_2 < t < t_3 = t_1^2 (1 + v/a). \end{cases} \tag{2.3}$$

Here

$$N_0 = R(\bar{v}, 0)/\sqrt{\pi v t},\quad I_0 = 1/\sqrt{vt},\quad I_1 = \frac{G(-a/c)\sqrt{a/c - 1}}{\sqrt{v(t_2 - t)}\sqrt{a/v - a/c}}, \tag{2.4}$$

$$I_2 = \frac{1}{2\pi\sqrt{vt}} \int_1^{a(t-1)/(vt)} \frac{\{G(-s)\}\sqrt{s-1}\, ds}{\sqrt{a(t-1)/(vt) - s(a/c - s)}},$$

$$m = \frac{\sqrt{2}(1 - v/c)G^{-1}(-a/v)}{\sqrt{\pi}\sqrt{1 - v/a}},\quad G(-s) = \exp\left[\frac{1}{\pi}\int_1^{a/b} \frac{\varphi(\xi)}{\xi - s}\, d\xi\right],$$

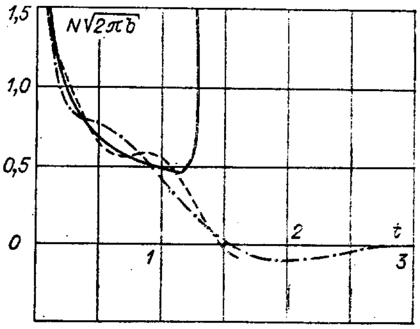

Fig. 2

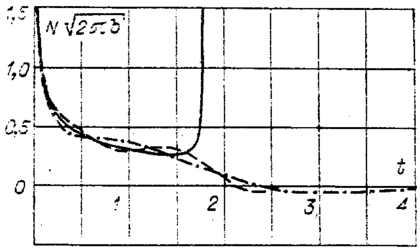

Fig. 3

$$\varphi(\xi) = \operatorname{arctg} \frac{4\xi^2 \sqrt{\xi^2 - 1} \sqrt{a^2/b^2 - \xi^2}}{(2\xi^2 - a^2/b^2)^2},$$

$$\{G(-s)\} = \cos(\varphi(s)) \exp(\varkappa(s)), \quad \varkappa(s) = \text{v.p.} \frac{1}{\pi} \int_1^{a/b} \frac{\varphi(\xi)\, d\xi}{\xi - s},$$

c is the rate of propagation of Rayleigh waves (with $b/a = 0.6$, $c/a = 0.5485$). The values of t_0, t_1, t_2 correspond to the moments of time of the arrival of the signals of longitudinal, transverse, and Rayleigh waves, emitted with t = 1 at the center of the cut, at the end of the cut. The value of t_3 corresponds to the moment of time of the arrival at one end of the cut of the signal of a longitudinal wave, emitted with t = 1 at the center of the cut and reflected from its other end.

The introduction of t_3 means that solution (2.3) is valid when there is no effect of one end of the cut on the other end.

A comparison of curves in Figs. 2 and 3 shows that the approximate solution of the polynomial approximation (2.2) does not reproduce the behavior of N(t) in the neighborhood of t_2, i.e., the time of arrival at the end of the cut of a Rayleigh wave emitted at the center of the cut at the moment when the action of the load ceases (t = 1), the remaining time qualitatively and, with $0 < t < t_1$ quantitatively, coincides with the exact value.

Figures 4-6 show dependences N(t), found using (2.2) for the law $\sigma(t) = \sigma_0(t)\delta_0(1 - t)\sin \pi t$, $0 < t < T$ with r = 9 and values of T = 1, 2, 4 (curve 1, and the dashed and dash-dot curves, respectively) for v/b = 0.2, 0.4, 0.6.

N(t) is frequently evaluated using the static approximation $N^\circ = \sigma(t)/\sqrt{\pi v t}$, where $N^\circ(t)$ for the moment of time t is the value of the coefficient of the stress intensity in the static problem of an isolated cut of length 2vt, loaded at the center by forces with a value of $\sigma(t)$.

This approximation is shown by curves 2 in Figs. 4-6. It can be seen that, even with v = 0.4 or greater, the difference between the dynamic solution and the static approximation is great, and the time of the positive phase of the functions N(t) and $N^\circ(t)$ differs particularly strongly.

In [1], for the case of antiplane deformation, that a good approximation of the dynamic problem for a growing isolated cut with large values of the rate of its development is a solution of the problem for a semifininite cut, which, at the initial moment of time, draws away from the origin of coordinates, and then develops at a constant rate v.

In this case the edges of the cut are loaded by forces concentrated at the origin of coordinates, varying with the time in accordance with the same law $\sigma(t)$ as in the problem for an isolated cut.

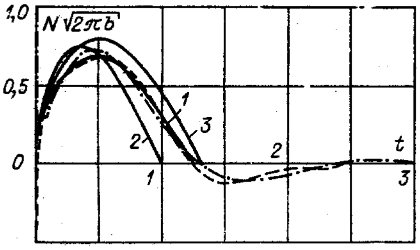

Fig. 4

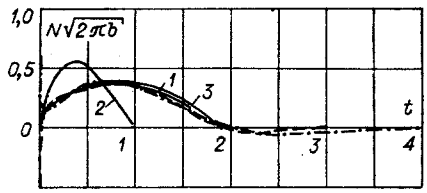

Fig. 5

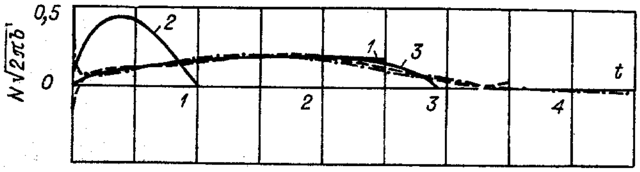

Fig. 6

The applicability of such an approximation is obviously explained by the weakening, with a rise in v, of the effect of one end of the crack on the other.

In accordance with [2], the expression of the coefficient of the stress intensity for the problem of a growing semiinfinite cut, loaded in the above manner, can be represented in the form

$$N^1(t) = m(I_0 - I_1 - I_2)/\sqrt{l(t)},\ I_0 = \sigma[t - l(t)/b], \tag{2.5}$$

$$I_1 = G(-a/c)\sqrt{a/c-1}\sqrt{(1/c-1/b)\,l(t)}\int\limits_0^{l(t)/a} \sigma'[t - l(t)/c - x(a/b - a/c)]\frac{dx}{\sqrt{x}},$$

$$I_2 = \frac{\sqrt{l(t)/a}}{\pi}\int\limits_1^{a/b}\int\limits_0^{l(t)/a} \sigma'[t - sl(t)/a - x(a/b-s)]\,\frac{\{G(-s)\}\sqrt{s-1}\sqrt{a/b-s}}{\sqrt{x}\,(a/c-s)}\,dxds, \tag{2.5}$$

where (t) is the path traversed by the tip of the cut after the start of its motion. In the case of a uniform development of the cut, $l(t) = vt$. The remaining notation in (2.5) is the same as in (2.4).

Calculations using formula (2.5) for $\sigma(t) = \delta_0(t)\delta_0(1-t)\sin\pi t$ and $l(t) = vt$ are shown in Figs. 4-6 by curves 3. There is good agreement between the dependences N(t) and $N^1(t)$ with $v \geqslant 0.4$.

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